MTH401 MID TERM PAST PAPERS (FILE PART II) SOLVED

BY MASOOM FAIRY

Note:

- **I could not make Neat File due to Much Load shedding.**
- **There is an other file because of Large size of this one.**

MTH401 Deferential Equations

Mid Term Examination – Spring 2006
Time Allowed: 90 Minutes

Question No. 1	Marks : 1

The method of undetermined coefficient is limited to homogeneous linear differential equation

- True
- False Page 148

The Method of Undetermined Coefficient

The method of undetermined coefficients developed here is limited to non-homogeneous linear differential equations

Question No. 2 **Marks : 1**

In the homogeneous differential equation after substitution v=y/x the equation reduces to.

- Separable differential equation.
- **Exact differential equation. Lecture 5**
- Remain homogeneous equation.
- None of the other

Question No. 7 **Marks : 1**

If the Wronskian W of three function f(x),g(x),h(x) is zero, what can be said about the dependency of the functions

- **May or may not be dependent** page 113
- Always dependent
- Never dependent
- None of the other

A Vanishing Wronskian does not guarantee linear dependence of functions.

Question No. 8 **Marks : 1**

If $a_n(x) = 0$ in the differential equation

$$a_n(x)\frac{d^n y}{dx^n} + a_{n-1}(x)\frac{d^{n-1}y}{dx^{n-1}} + a_{n-2}(x)\frac{d^{n-2}y}{dx^{n-2}} + \dots + a_1(x)\frac{dy}{dx} + a_0(x)y = g(x)$$

for some $x \in I$ then

I. Solution of initial value problem may not unique.
II. Solution of initial value problem may not even exist.
III. Solution of initial value problem should exist.
IV. Solution of initial value problem is unique.

- I is correct only
- I and II are correct
- I and III are correct
- IV is correct only

Question No. 9 **Marks : 1**

Equation of the form $\frac{dy}{dx} + y = x^2 y^2$ is called

- First order linear differential equation
- Bernoulli equation
- Separable equation

❺ **None of the other.**
According to all above equations.

FINALTERM EXAMINATION
FALL 2006
MTH401 - DIFFERENTIAL EQUATIONS (Session - 1)

Q: 1: If the variation of the path of the curves can be described by the concept of differential equations then which of the following differential equation describe the path for $y\ axis$.

► $\frac{dy}{dx} = 1$

► $\frac{dy}{dx} = 0$ **Not confirm**

► $\frac{dy}{dx} = -1$

► $\frac{dy}{dx} = \infty$

Q: 2: Suggestive form of the constant input function for the non homogeneous differential equation under the method entitled as "**Method of the undetermined coefficient**" is

1 $f(x) = e^{x}$

2 $f(x) = a$

3 $f(x) = e^{ax}\left(ACosx + BSinx\right)$

4 **Suggestive form is impossible. PAGE 148**

Q: 3: Which of the following function is linearly dependant to the exponential function e^x ?

- ▶ $-e^x$
- ▶ e^{-x} ***not confirm***
- ▶ xe^x
- ▶ $-xe^{-x}$

Q: 4: Eigen values for the system of the differential equations $X = AX$ are evaluated for the

- ▶ Solution vector X
- ▶ Coefficient matrix A
- ▶ Differentiated solution vector X
- ▶ Transpose of the Coefficient matrix A

Q: 5: Fundamental set of the solution vectors $X_1, X_2, \ldots, X_n$ for any system of the differential equations are obtained by

- ► **Developing the singleton set $\{X\} = \{c_1 X_1, c_2 X_2, \ldots\ldots, c_n X_n\}$ of the linear combinitions of the solution vectors.** **Page 389**

- ► Taking derivative of the each solution vector and forming the set

X' X' X'

- Taking Integral of the each solution vector and forming the set $\left\{\int X_1 dx, \int X_2 dx, \cdots\cdots, \int X_n dx\right\}$

- Just verifying their linear independance and establishing the set $\left\{X_1, X_2, \cdots\cdots, X_n\right\}$

MID TERM EXAMINATION
SPRING 2007
MTH401_ SESSION 4

Question No: 1 (Marks: 1) - Please choose one
The differential equation $(3x^2 y + 2)dx + (x^3 + y)dy = 0$ is

PAGE 26

- **Exact**
- Linear
- Homogenous
- Separable

Question No: 2 (Marks: 1) - Please choose one

The assumed particular solution for the U.C(Undetermined Coefficient) differential equation $y' - y = x^2 e^{2x}$ is

- $y_p = c_1 e^{x_2} + c_2 x^2$
- $y_p = (Ax + B)e^{2x}$
- $y_p = (Ax^2 + Bx + c)e^{2x}$
- None of these.

Question No: 3 (Marks: 1) - Please choose one

The differential equation $x\frac{dy}{dx} + y = y^2 \ln x$ is an example of

- Separable
- Homogenous
- Exact
- **None of these.**

Question No: 4 (Marks: 1) - Please choose one

For the differential equation

$y' - 2xy = x$

Integrating factor is

► x^2

PAGE 34

► e^{x^2}

► e^{x^2}

► x^2

Question No: 5 (Marks: 1) - Please choose one

$$\frac{dy}{dx} = \frac{x+3y-5}{x-y-1}$$

Identify the ordinary differential equation

► Homogenous

► Separable

► **Exact** **PAGE 26**

► None of these.

MIDTERM EXAMINATION
(Solution File)

SEMESTER SPRING 2004
MTH401- Differential Equations

Q: 1: The differential equation $\sec y \frac{dy}{} + \sin(x-y) = \sin(x+y)$ is

➔ **Separable** **PAGE 7**

Q: 2: The integrating factor of the differential equation $(x^2+1)\frac{dy}{dx} + 2xy = 1$ is

➔ **x^2 +1** PAG 34

Q: 3: The form of the particular solution for the differential equation

$$y'' - y = x^4$$

➔ $y_p = A_4 x^4 + A_3 x^3 + A_2 x^2 + A_1 x + A_0$

Q: 4: Determine which of the given functions are linearly independent.

➔ $f_1(x) = 1 + x, f_2(x) = x, f_2(x) = x^2$

PAGE 110

Q: 5: The differential operator that annihilates $10x^3 \; 2x$ is:

➔ D^4

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Question No: 6 **Marks:10**

Solve the following differential equation by using an appropriate substitution.

$$\frac{dy}{dx} = \frac{y}{x} + \frac{x}{y}$$

Solution

$$\frac{dy}{dx} = \frac{y}{x} + \frac{x}{y}$$

$$\frac{dy}{dx} = \frac{y^2 + x^2}{xy}$$

Homogeneous equation, so put $y = vx, \frac{dy}{dx} = v + x\frac{dv}{dx}$

$$v + x\frac{dv}{dx} = \frac{v^2x^2 + x^2}{x^2v}$$

$$v + x\frac{dv}{dx} = v + \frac{1}{v}$$

$$x\frac{dv}{dx} = \frac{1}{v} \qquad vdv = \frac{1}{x}dx$$

$$\int vdv = \int \frac{1}{x}dx \qquad \frac{v^2}{2} = \ln x + \ln C$$

$$\frac{y^2}{x^2} = 2\ln xC$$

Question No: 7 **Marks:10**

The population of a town grows at a rate proportional to the population at any time. Its initial population of 500 increases by 15% in 10 years. What will be the population in 30 years?

Solution:

Let $P(t)$ be the population at any time t, then rate of grows will be

---

$$\frac{dP}{dt} P$$

$$\frac{dP}{dt} = kP$$

Here k is constant of proportionality. Since initially population was 500, therefore $P(0) = 500$. Also this population increases by 15% in 10 years. The 15% of 500 is $\frac{15}{100}(500) = 75$, therefore population after 10 years is (initial population + increase in 10 years) = 500+75 = 575 i.e. $P(10) = 575$. So we have the boundary value problem

$\frac{dP}{dt} = kP$ subject to boundary conditions $P(0) = 500$, $P(10) = 575$.

This first order differential equation. Its solution is given by

$P = Ce^{kt}$ where C is constant of integration.

Applying boundary conditions, we get $C = 500$, $k = 0.0139$. So the solution is

$$P(t) = 500e^{(0.0139)t}$$

Thus population after 30 years is obtained by putting t =30 in above equation i.e.

$$P(30) = 500e^{(0.0139)30}$$
$$\approx 760.$$

Question No: 8 **Marks:10**

Find a second solution of following differential equations where the first solution is given. You can use any method (reduction of order or formula given in handouts).

$$x^2 y'' + 2xy' - 6y = 0; \quad y_1 = x^2$$

MIDTERM EXAMINATION (Solution File)

SEMESTER SPRING 2004

MTH401- Differential Equations

Question No: 1 **Marks: 2**

The differential equation $\frac{dy}{dx}=\frac{x+3y}{3x+y}$ is

Homogeneous

Question No: 2 **Marks: 2**

The integrating factor of the differential equation $\frac{dy}{dx}-y=e^{3x}$ is

e^{x}

Question No: 3 **Marks: 2**

The form of the particular solution for the differential equation

$y'-y=\cos 2x$

$y_p = A\cos 2x + B\sin 2x$ *repeat*

Question No: 4 **Marks:2**

Determine which of the given functions are linearly independent.

$f_1(x)=x, f_2(x)=x^2, f_2(x)=4x-3x^2$ **Repeated**

Question No: 5 **Marks:2**

The differential operator that annihilates $4e^{x/2}$ is:

$2D-1$

Question No: 6 **Marks:10**

Solve the following differential equations.

$$\left(1+\ln x+\frac{y}{x}\right)dx=(1-\ln x)dy$$

Solution:

Here

$$M=1+\ln x+\frac{y}{x},\quad N=-(1-\ln x)$$

$$M_y=\frac{\partial M}{\partial y}=\frac{1}{x},\qquad N_x=\frac{\partial N}{\partial x}=\frac{1}{x}$$

$$M_y=N_x$$

So the given equation is an exact equation. Thus there exists a function $f(x,y)$ such that

$$\frac{\partial f}{\partial x}=M \quad and \quad \frac{\partial f}{\partial y}=N$$

$$\frac{\partial f}{\partial x}=1+\ln x+\frac{y}{x}\ --(1) \quad and \quad \frac{\partial f}{\partial y}=\ln x-1\ --(2)$$

$$(1)\ f=x+x\ln x-x+y\ln x+H(y)=x\ln x+y\ln x+H(y)\quad \frac{\partial f}{\partial y}=\ln x+H'(y)$$

$$(2)\ \ln x-1=\ln x+H'(y)$$

1. $-1=H'(y)$
2. $H(y)=-y$

$$Hence\quad f(x,y)=x\ln x+y\ln x-y$$

Initially there were 100 milligrams of a radioactive substance present. After 6 hours the mass decreased by 3%. If the rate of decay is proportional to the amount of the substance present at any time, find the amount remaining after 24 hours.

Solution:

Let $A(t)$ be amount present at any time t. Then by given conditions, we have

$$\frac{dA}{dt}$$

$$A \quad \frac{dA}{dt} = kA$$

Initially there were 100 milligrams, therefore $A(0)=100$. Moreover, decreased by 3% will give us $100 - \frac{3}{100}(100) = 97$ milligrams after 6 hours i.e. $A(6) = 97$. So we have boundary value problem

$$\frac{dA}{dt} = kA \text{ subject to boundary conditions } A(0)=100, A(6)=97$$

The solution of this equation is given by

$A(t) = Ce^{kt}$ where C is constant of integration.

Applying boundary conditions, we get

$$C = 100, \quad k = -0.005076$$

$$A(t) = 100e^{-0.005076t}$$

Amount remaining after 24 hours is obtained by putting $t = 24$ in above equation i.e.

2. $A(t) = 100e^{-0.005076(24)}$

$1\,88.529 \ mg.$

Question No: 8 **Marks:10**

Find a second solution of following differential equations where the first solution is given. You can use any method (reduction of order or formula given in handouts).

$$x^2 y'' + y' = 0; \quad y_1 = \ln x$$

Solution:

Comparing this equation with $y'' + P(x)y' + Q(x)y = 0$, we get

$$P(x) = \frac{1}{x^2}.$$

But second solution is given by

$$y_2 - y_1 \int \frac{e^{-\int P(x)dx}}{y_1^2} dx$$

1

MIDTERM EXAMINATION (Solution File)

SEMESTER SPRING 2004

MTH401- Differential Equations

Question No: 1 **Marks: 2**

The differential equation $(x+y)(x-y)dx + x(x-2y)dy = 0$ is

Exact **PAGE 26**

Question No: 2 **Marks: 2**

The integrating factor of the differential equation $(2y^2+3x)dx + 2xydy = 0$ is

x ***not confirm***

Question No: 3 **Marks: 2**

The form of the particular solution for the differential equation

$y'' - y = \cos x + e^x$ is:

$y_p = Ae^x + B\cos x + C\sin x$ ***Repeated***

Question No: 4 **Marks:2**

Determine which of the given functions are linearly independent.

$f_1(x) = x, f_2(x) = x^2, f_2(x) = 4x - 3x^2$ **REPEATED**

The differential operator that annihilates $4e^{2x}$ is:

$(D-2)(D+5)$

Question No: 6 **Marks:10**

Find the general solution of the given differential equation.

$$\frac{dy}{dx} + 2xy = x^3$$

Solution:

It is of the form $\frac{dy}{dx} + P(x)y = Q(x)$ i.e. Linear First Order Differential Equation with

$P(x) = 2x,\ Q(x) = x^3$

Thus integration factor is given by

$$I.F = u(x) = e^{\int P(x)dx}$$

$$= e^{\int 2xdx} = e^{x^2}$$

But the solution in this case is

$$y = \frac{\int u(x)Q(x)dx + C}{u(x)} \quad \text{-------------------(1.}$$

Now

$$\int u(x)Q(x)dx = \int x^3 e^{x^2}$$

$$= \frac{1}{2}\int (e^{x^2} 2x)x^2 dx$$

$$= \frac{1}{2}\left\{e^{x^2}x^2 - \int e^{x^2} 2xdx\right\} \quad int\ egration\ by\ parts$$

$$= \frac{1}{2}\left\{e^{x^2} x^2 - e^{x^2}\right\}$$

So the solution is

$$y = \frac{\frac{1}{2}\{x^2 - 1\} e^{x^2} + C}{e^{x^2}}$$

$$= \frac{1}{2}(x^2 - 1) + Ce^{x^2}$$

Question No: 7 **Marks:10**

A thermometer is taken from an inside room to the outside where the air temperature is $5°F$. After 1 minute the thermometer reads $55°F$, and after 5 minutes the reading is $30°F$. What is the initial temperature of the room?

Solution:

Let $T(t)$ be temperature at any time t and T_0 be the temperature of the surroundings. Then by Newton's Method, we know that

$$\frac{dT}{dt} = k(T - T_0)$$

Where k is constant of proportionality. Here we are given $T_0 = 5$ and $T(1) = 55, T(5) = 30$. Solving above equation we get

$$T = T_0 + Ce^{kt}$$

$$7 \quad T = 5 + Ce^{kt}$$

Using above conditions we get

$$k = -0.173, C\ 59.44.$$

So the initial temperature is given by

$$\blacktriangleright \quad = 5 + Ce^0$$

$$5 + C$$

$$5 + 59.44 = 64.44^o F.$$

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